# A NEW NONLINEAR TRANSFORM FOR IFS COMPRESSION OF ECG AND OTHER SIGNALS

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*Abstract*-**This paper presents a nonlinear extension of the affine transform as used in the iterated function systems (IFS) to compress signals and data. Compared with ∅ien and Nårstad's orthogonal transform with compression ratio of 6.0:1 for the electrocardiogram (ECG) signal, the nonlinear approach produces higher compression ratios of 10.2:1, and is more flexible in finding the corresponding strange attractor of the ECG signal. It can model the QRS complex of the ECG signal well, which has been a problem for the affine transform in fractal compression.**

*Keywords*-**ECG, strange attractor, IFS, affine transform.**

## I. INTRODUCTION

Data compression based on fractals commenced about ten years ago [1]. More specifically, the iterated function systems (IFS) approach to data compression has been developed based on the fundamental property of fractal objects: self-similarity. Thus, our discussion begins from a fractal signal. A random discrete process $x(n)$ defined for all integers, $-\infty < n < \infty$, is said to be statistically self-similar if its statistics are invariant to its dilations and contractions. Thus, $x(n)$ is statistically self-similar with a parameter $D$ if for any real $\alpha > 0$, it obeys the following scaling relation [2]

$$x(n) \overset{p}{=} \alpha^{-D} x(\alpha n) \tag{1}$$

where $\overset{p}{=}$ denotes equality in a statistical sense.

To employ the multiplicative (rather than additive) invariance of the dilation and contraction of the fractal signal, Barnsley proposed a fractal compression technique, the IFS, which uses an affine transform to map the process onto itself [1]. Such an affine transform, $W$, for a one-dimensional process $x(n)$ is defined as

$$W\begin{pmatrix} n \\ x \end{pmatrix} = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix} \begin{bmatrix} n \\ x \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \tag{2}$$

where $a$, $c$, $d$, $e$, and $f$ are the transform coefficients.

The IFS compression technique partitions the process into domain blocks and range blocks, and finds a set of affine transforms between the domain blocks and range blocks. Computational complexity, compression ratio, reconstruction quality of the signal, and convergence are important issues in the IFS approach [1]. All these issues are related to the quality of the matching between the blocks which results in the affine transform. The role of the affine transform is to fit the range blocks with the domain blocks optimally. The compression ratio and reconstruction quality are decided by the optimization algorithm of the affine transform. The current transform in the IFS described by (2) is a linear affine transform. Its fitting capability is not satisfactory for signals such as image, speech, and electrocardiogram (ECG), which have nonlinear characteristics. In addition, the original IFS search algorithm is very time consuming due to the direct comparison of a large number of domain blocks [3]. An order of magnitude improvement in the search time was achieved through a frequency-sensitive neural-network search [4] [5]. It is necessary to find another model for the IFS to improve compression performance and reduce the search time.

A more flexible way to model complicated signals is to use a nonlinear transform. We use a polynomial expansion to model the ECG signal because it is an efficient way to model arbitrary functions. Convergence is an important issue in the signal reconstruction of the IFS for the proposed transform. Although we have not proved it in theory, our experiments show that the new extended affine transform gives convergent results. This nonlinear model also demonstrates the flexibility in modelling various complexities in the ECG signal. We shall show that it can give much better reconstruction quality under the same compression ratio, as compared with a corresponding linear model. The difficulty in modelling the QRS complex in ECG signals [6] is solved with the proposed technique. The new transform combined with complexity measure can reduce the computational complexity [7]. Experimental results are presented and compared with the standard IFS and other techniques.

For clarity and completeness, a continuous ECG signal is first obtained from multi-channel sensors, then amplified and filtered to satisfy the Nyquist sampling theorem, then

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**Abstract**

**Subject Terms**

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sampled to form a discrete signal, and finally quantized to form a digital ECG signal. This paper is concerned with the digital ECG form only.

## II. The Nonlinear IFS (NIFS)

The IFS data compression is realized through finding a set of affine transforms between range pool and domain pool. The role of the affine transform is to fit specific range blocks with the domain blocks optimally. The compression performance is directly decided by the capability of the affine transform that models the signal. The current transform used in the IFS is a linear affine transform. It may model perfectly some signals which have strict inherent linear relationship (such as the Cantor set, Koch curve, Sierpinski carpet, and Julia set) [8]. Unfortunately, many signals in real life have relationships that are more complicated than linear. To model such signals, the linear affine transform is inadequate. Image compression by the IFS fractal method is an example of how difficult it is to find a suitable mapping between a range block and the domain blocks. A complicated scheme that requires a point-by-point search combined with a variable size of the domain blocks is necessary to find a reasonable mapping, even though it may fail occasionally. This transform causes two problems: (i) poor compression performance and (ii) a very time-consuming search.

Instead, a generalized transform which may represent an arbitrary function is needed to model such complicated signals. For a one-dimensional signal $x(n)$, a generalized transform is defined as

$$W\begin{pmatrix} n \\ x \end{pmatrix} = \begin{bmatrix} an + e \\ g(x, n) \end{bmatrix} \tag{3}$$

where the coefficient $a$ is limited to the range $[0, 1)$ to guarantee contraction of signals in time. If $g(x, n)$ is also a contractive transform, then (3) satisfies the convergence condition of the IFS.

To find an appropriate $g(x, n)$, it is not sufficient to use the convergence condition only. According to Taylor's remainder formula, a function may be expanded as a $k$th order polynomial format if its first $k - 1$ derivatives are continuous and its $k$th derivative exists [9]. Thus, a $k$th-order polynomial is used to approximate the $g(x, n)$

$$g(x, n) \approx \sum_{p=0}^{k} \sum_{q=0}^{p} b_{pq} x^q n^{p-q}, \quad \forall k, p, q \in Z \tag{4}$$

where coefficients $b_{pq}$ is decided by the function $g(x, n)$ according to Taylor's remainder formula. Since $g(x, n)$ is unknown, it may be determined with the inverse procedure by finding the $b_{pq}$. The definition of the strange attractor of objects gives a way of how to find the $b_{pq}$. To obtain the strange attractor of the object, it is reasonable to assume [1]

$$g(x(n_1), n_1) = x(n_2) \tag{5}$$

where $x(n_1)$ is mapped to $x(n_2)$ under the condition

$$an_1 + e = n_2 \tag{6}$$

Equations (4), (5), and (6) constitute our nonlinear contraction transform for finding the strange attractor of the object. When the expansion order $k$ is equal to 1, the nonlinear transform becomes the traditional affine transform. Therefore, one sees that the nonlinear transform may lead to more flexible mappings between the range blocks and the domain blocks than the linear affine transform. Such flexibility may result in two advantages: (i) improving compression performance and (ii) speeding up the search.

We shall now discuss how to use the least square error technique to determine the coefficients in (4) and (6). For a lossless fractal compression technique, a transform must be found to satisfy (5) with a measured signal exactly. In practice, it is reasonable to use an approximation to replace the strict equality condition between $g(x, n)$ and $x(n)$ in the lossy fractal compression. Then there is an error

$$E = \sum_{n_2=1}^{r_l} [g(x(n_1), n_1) - x(n_2)]^2 \tag{7}$$

where $r_l$ is the length of the range block.

A set of optimal transform coefficients can be obtained by applying the least square error approach on (7).

## III. Experimental Results and Analysis

The new nonlinear IFS transform is applied to compress a one-dimensional ECG signal. To compare compression performance, the traditional IFS technique is also implemented. The performance of a compression algorithm is dependent mainly on two parameters: compression ratio and reconstruction (distortion) error. The compression ratio, *Rcr*, is defined as [10]

$$Rcr = \frac{\sum_{n=1}^{N} size[x_{ori}(n)]}{\sum_{n=1}^{N_1} size[x_{cmr}(n)]} \tag{8}$$

where $x_{ori}(n)$ and $x_{cmr}(n)$ represent the original and the compressed signal, respectively. Their corresponding sample lengths are $N$ and $N_1$.

A common distortion measure is the following percent root-mean-square difference (*PRD*) to measure reconstructed error of signals [11]

$$PRD = \sqrt{\frac{\sum_{n=1}^{N}[x_{ori}(n) - x_{rec}(n)]^2}{\sum_{n=1}^{N} x_{ori}^2(n)}} \times 100\,\% \quad (9)$$

where $x_{rec}(n)$ represents the reconstructed signal. In the processing of reconstruction error, it is found that the *PRD* will change when the signals shift along amplitude direction. A normalized *PRD* measure is proposed in [12] for ECG signals

$$NPRD = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[\frac{x_{ori}(n) - x_{rec}(n)}{max(x_{ori}) - min(x_{ori})}\right]^2} \times 100\,\% \quad (10)$$

Notice that although (9) is not a good error measure in ECG data compression, it is used in this paper to measure reconstruction error for convenience to compare our results with other research.

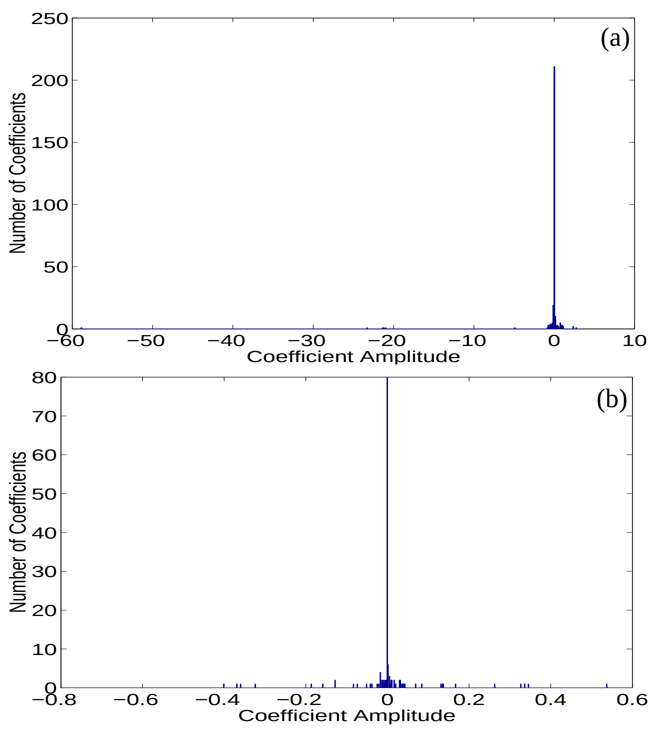


Fig. 1 Distribution of coefficients (a) $b_{10}$ and (b) $b_{20}$.

Prior to the calculation of the compression ratio and reconstruction error, the coefficients in $g(x, n)$ must first be quantized. Figure 1 shows two distributions of coefficients $b_{10}$ and $b_{20}$ with the ECG signal sampled at 360 Hz with 11 bit resolution, as taken from the MIT-BIH ECG database [13]. We do experiment using the ECG data contained in the file, x_100.txt, which has a 10-minute recording of the ECG signal. The Lloyd-Max nonuniform quantizer is used to quantize the coefficients because their distribution is close to a Laplacian [14].

To improve compression performance, the coefficient $a$ is set to $2^{-m}$ where $m = 1, 2, 3$, to increase the chance for finding the optimal mapping. The length of range blocks is also a variable, changing according to $2^l$ with the integer $l$ varying from 2 to 6. Figures 2(a) and (b) give the size distributions of range blocks for the traditional and the nonlinear IFS (NIFS) approach, respectively. The NIFS can use the most frequent range block size of 32 sample points to model the ECG signal compared with 4 sample points used by the linear affine transform. The size distributions of range blocks show that more large range blocks are found by the NIFS than by the traditional IFS, thus leading to an improved compression ratio. In our experiment, the mean range size is about 30.0 for the NIFS and 26.9 for the IFS. It demonstrates that the nonlinear transform can model signals more flexibly.

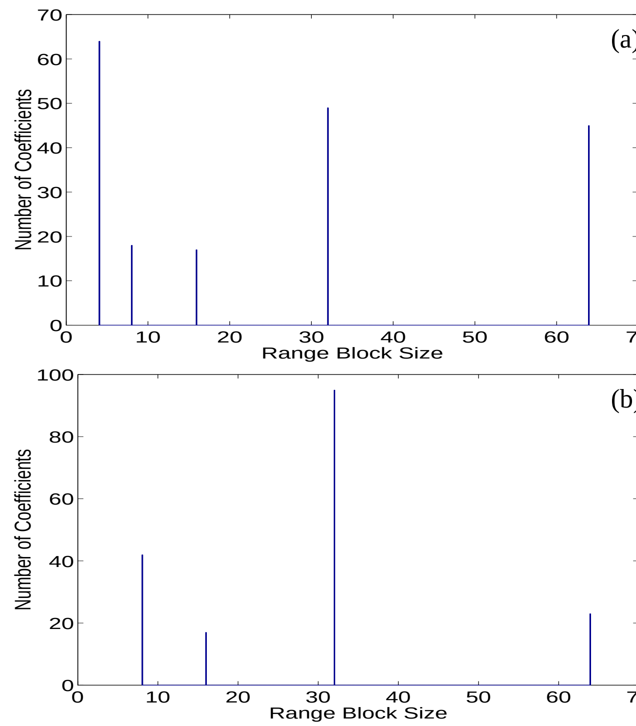


Fig. 2 Range block size distributions for (a) the traditional affine transform and (b) the nonlinear transform.

Figures 3(a) and (b) give address distribution of the optimally mapped domain blocks found by the traditional IFS and the NIFS approach, respectively. The address distribution of the mapped blocks given by the NIFS demonstrates more concentrated distribution around the first point of the search. Compared with full distribution in the segment of

1024 points of the IFS, the NIFS gives distribution of domain block address within 110 point range. It shows that the optimal mapping can found by searching fewer domain blocks. This can be useful in developing a fast search algorithm in the future.

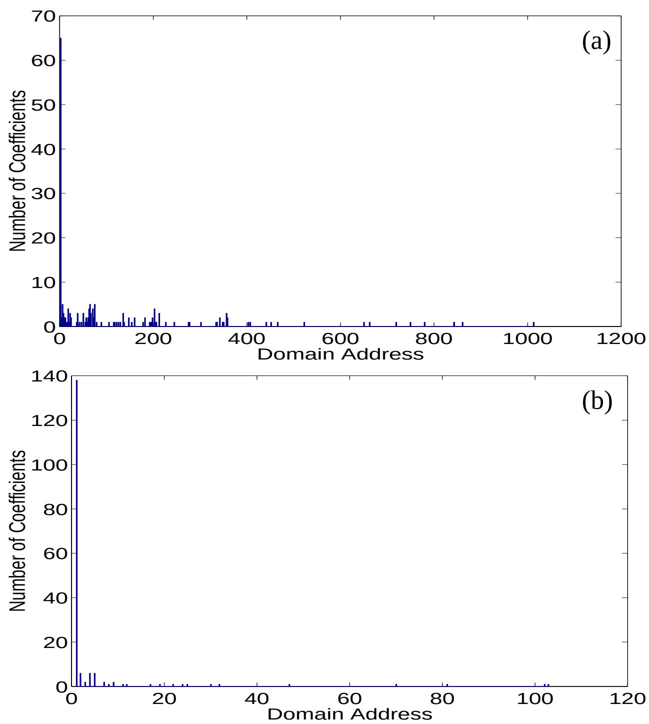


Fig. 3 Address distribution of the optimally mapped domain blocks found by (a) the traditional and (b) the NIFS approach.

Figures 4(a), (b), and (c) show the original ECG signal, its reconstruction, and the reconstructed error. From these figures, one can see that the NIFS compresses and reconstructs the ECG signal extremely well. The new technique has the remarkable property of modelling the QRS complex of the ECG signal almost perfectly, which is a difficult issue for the linear affine transform and the orthogonal transform [6]. The reconstruction error is limited to a small range only. It is mainly due to the noise in the signal.

Table 1 shows the influence of the order *k* of the Taylor series expansion and quantization resolution on the compression performance of the ECG signal. By changing the expansion order from 1st to 4th, as well as the resolution of the quantizer from 7 bits to 14 bits, various *PRD* and *Rcr* are obtained. By monitoring the changes among them, optimal parameters for the NIFS on the ECG signal can be found.

Table 1 shows that when the resolution increases, the *Rcr* increases. The *PRD* is controlled by the error threshold. When quantizer takes more than 11 bits, the *PRD* and the *Rcr* change little with the resolution increasing further. In the future, we will use different quantizer for different coefficient.

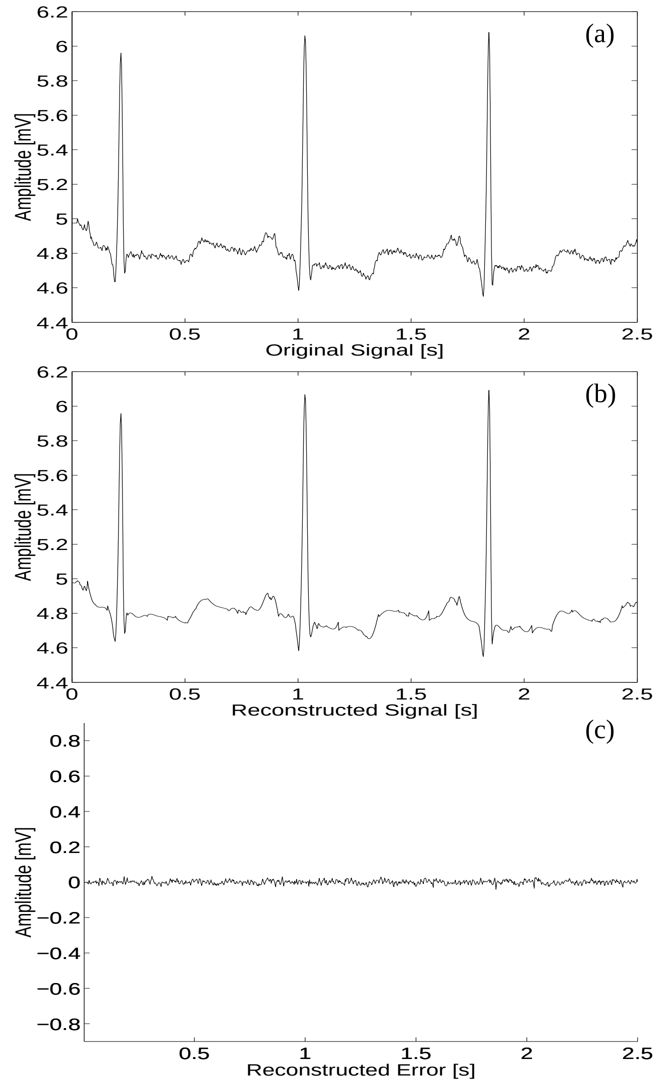


Fig. 4 The ECG signal compression: (a) original ECG signal, (b) reconstructed signal, and (c) reconstructed error.

Table 1: Compression performance investigation of the NIFS with quantization resolution and polynomial expansion change on the ECG signal.

| Resolution (bit) | k=1 | | k=2 | | k=3 | | k=4 | |
|---|---|---|---|---|---|---|---|---|
| | PRD | Rcr | PRD | Rcr | PRD | Rcr | PRD | Rcr |
| 7 | 6.0% | 9.1 | 5.8% | 6.0 | 5.6% | 3.0 | 5.5% | 2.2 |
| 8 | 5.8% | 9.0 | 5.4% | 5.4 | 5.7% | 4.9 | 5.7% | 4.0 |
| 9 | 5.8% | 9.5 | 5.7% | 8.0 | 5.6% | 6.1 | 5.8% | 5.3 |
| 10 | 5.7% | 10.5 | 5.6% | 7.6 | 5.3% | 6.0 | 5.8% | 6.1 |
| 11 | 5.8% | 10.6 | 5.9% | 9.9 | 5.8% | 7.9 | 5.9% | 7.4 |
| 12 | 5.8% | 10.7 | 5.8% | 9.9 | 5.9% | 8.6 | 5.8% | 7.7 |
| 13 | 5.9% | 10.6 | 5.8% | 10.2 | 6.0% | 9.2 | 6.0% | 8.3 |
| 14 | 5.8% | 11.1 | 5.8% | 10.2 | 5.7% | 9.2 | 5.4% | 8.4 |
| Threshold | 0.0115 | | 0.0117 | | 0.0123 | | 0.0130 | |

One may predict that the expansion order increase will result in a *PRD* decrease because of the more powerful modelling ability of the NIFS. However, a higher order results in more coefficients, thus leading to a lower compression ratio.

Consequently, there must be a trade-off between the *RMS* and the *Rcr* for choosing the optimal order *k*. Although Table 1 demonstrates that the change of expansion order has almost no influence on the *PRD* (because the *PRD* is controlled by the error threshold), it results in different *Rcr*. The expansion order of 2 may compete with the compression ratio achieved by the linear IFS.

It was found that the NIFS achieves almost the same compression ratio as the traditional IFS under the same reconstruction error in the ECG signal compression. When $PRD = 5.8\,\%$, ∅ien and Nårstad achieved 6.0:1 with their IFS ECG compression by orthogonal transform [6]. With the optimal choice of order 2 and 14 bit nonuniform quantizer, we get the compression ratio about 11.1:1 for the affine transform and about 10.2:1 for the NIFS.

## IV. Conclusion

In this paper, we use a nonlinear IFS (NIFS) transform to compress a one-dimensional ECG signal. The first important issue in this technique is the convergence of the strange attractor reconstruction. We have seen that the signal reconstruction is successful. The experiment shows that the nonlinear approach may compete with the traditional IFS technique for the ECG signal compression. With the optimal choice of order and nonuniform quantizer, the extended transform achieves a maximal compression ratio of 10.2:1, which is higher than that of 6.0:1 obtained by ∅ien and Nårstad under the same *PRD*.

Another advantage of the new extended transform is that it can model the QRS complex of the ECG signal very well, which has been a problem for the affine transform and the orthogonal transform in fractal compression. Fewer blocks are used to model the ECG signal by the NIFS than that by the linear IFS. It reveals that the NIFS technique has a more powerful modelling ability.

The address distribution of the optimally mapped domain blocks around the first point in the search in Fig. 3(b) has important significance. It can be employed to speed up the search of finding optimal mappings between the range blocks and domain blocks. In another paper, we describe how to use the extended transform to develop a fast NIFS algorithm.

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